mass separation of disparate mass particles. When these problems are minimized, the error can be 5% for relative concentrations of major species. This is the same level of accuracy that can be achieved by most shock tunnel instrumentation and is superior to other methods of species concentration measurements. At a cost much less than that of most optical systems, the time-of-flight mass spectrometer is a viable addition to the range of diagnostic tools for short duration aerodynamic testing.

Acknowledgments

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Lamination Parameters for Reissner-Mindlin Plates

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Introduction

TSAI and Pagano[1] introduced 12 parameters—called lamination parameters—to express plate stiffnesses of laminated composite Kirchhoff plates. These parameters considerably simplify design and optimization of thin composites compared with using layup angles and thicknesses of discrete plies in a laminate. A layup gives a unique set of lamination parameters, but many layups might give the same set of lamination parameters. The lamination parameters fully describe the plate stiffnesses, and they can therefore be used as design variables for structural design and optimization. Examples can be found in Miki[2,3] and Grenestedt.[4] By using these parameters rather than ply layup angles and thicknesses, the major problem of local optima during optimization is often overcome. The stiffnesses are linear in the lamination parameters, the range of the lamination parameters is convex (see Grenestedt and Gudmundson[5]), and a number of optimization problems therefore become convex (see Svanberg[6]). Further, all physically possible layups are simply included in the analysis with just a small number of parameters. The purpose of the current Note is to derive lamination parameters for kinematically nonlinear small strain, medium rotation shear deformable Reissner-Mindlin type[7,8] plates. For the sake of completeness, the Reissner-Mindlin equations as well as the Tsai and Pagano lamination parameters are derived in the following sections.

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Governing Equations of Laminated Reissner-Mindlin Type Plates

In the following, Einstein's summation convention is used. Greek indices run from 1 to 2 and Latin from 1 to 3. Cartesian Lagrangian coordinates x_i with the x_3 axis perpendicular to the undeformed plate middle surface are introduced. The displacement field is assumed to be

$$\begin{aligned} u_\alpha(x_1, x_2, x_3) &= \bar{u}_\alpha(x_1, x_2) + x_3\theta_\alpha(x_1, x_2) \\ u_3(x_1, x_2, x_3) &= \bar{u}_3(x_1, x_2) \end{aligned} \tag{1}$$

where u_i are displacement components, and $\bar{u}_i$ are middle surface displacement components. The Green-Lagrange strain tensor is, when gradients of in-plane displacements are assumed small,

$$e_{\alpha\beta} = \overset{0}{e}_{\alpha\beta} + x_3\kappa_{\alpha\beta} \tag{2}$$

where

$$\begin{aligned} \overset{0}{e}_{\alpha\beta} &= (\bar{u}_{\alpha,\beta} + \bar{u}_{\beta,\alpha} + \bar{u}_{3,\alpha}\bar{u}_{3,\beta})/2 \\ \kappa_{\alpha\beta} &= (\theta_{\alpha,\beta} + \theta_{\beta,\alpha})/2 \end{aligned} \tag{3}$$

and

$$e_{\alpha 3} = (\bar{u}_{3,\alpha} + \theta_\alpha)/2 \tag{4}$$

As constitutive relation, elastic behavior with a strain energy per unit plate surface area

$$W\left(\overset{0}{e}_{\alpha\beta}, e_{\alpha 3}, \kappa_{\alpha\beta}\right) \tag{5}$$

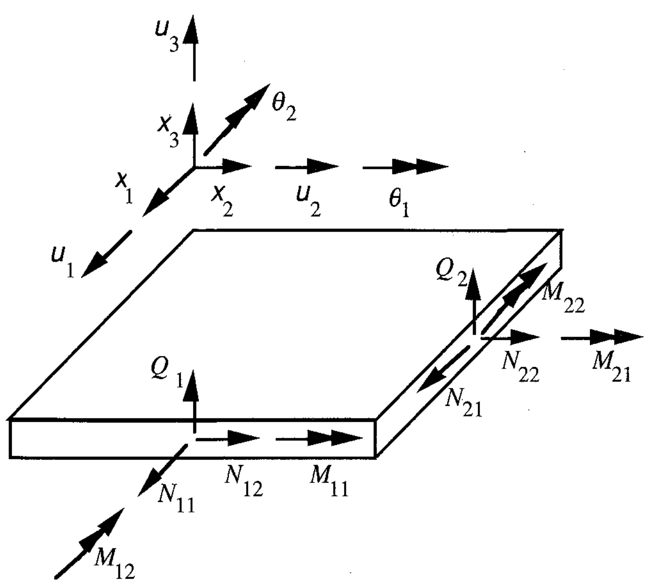

Fig. 1 Notation and sign conventions for the plate variables.

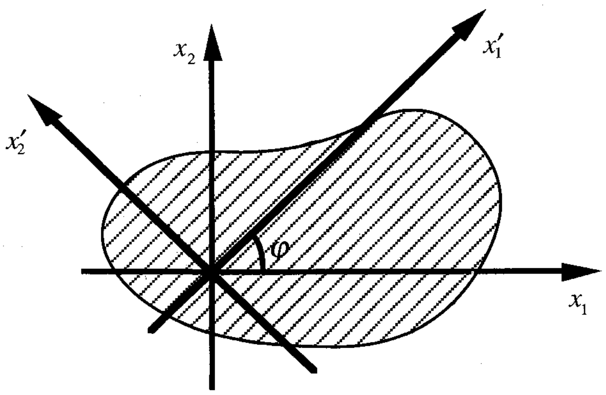

Fig. 2 Coordinates and ply orientation.

is assumed. Forces and moments per unit length, conjugate to strains and curvatures, are

$$N^{\alpha\beta} = \frac{\partial W}{\partial e^0_{\alpha\beta}}, \qquad 2Q^{\alpha} = \frac{\partial W}{\partial e_{\alpha 3}}, \qquad M^{\alpha\beta} = \frac{\partial W}{\partial \kappa_{\alpha\beta}} \tag{6}$$

See also Fig. 1. It is common in composite material mechanics to use a linear relation between Cauchy (true) stress and infinitesimal strain of the material in a single point of a ply. When small strain and arbitrary rotation are assumed, second Piola-Kirchhoff stress equals back-rotated Cauchy stress, and Green-Lagrange strain equals infinitesimal strain for the deformation without rotation. A full three-dimensional linear constitutive relation between second Piola-Kirchhoff stress T^{ij} and Green-Lagrange strain is thus assumed:

$$T^{ij} = C^{ijkl} e_{kl} \tag{7}$$

where because of the symmetry relations $C^{ijkl} = C^{jikl} = C^{klij}$ there are in the most general setting 21 independent elastic moduli. This Note concentrates on common composite material reinforcements such as unidirectional and fabric, for which some simplifications are appropriate. Assuming that the plane perpendicular to the x_3 axis is a plane of material symmetry, the following moduli are zero,

$$C^{1113} = C^{1123} = C^{2213} = C^{2223} = C^{3313} = C^{3323} = C^{1213} = C^{1223} = 0 \tag{8}$$

independent of rotation about the x_3 axis. By assuming that one more plane, perpendicular to the x_1' axis in Fig. 2, of material symmetry exists, the ply becomes orthotropic, and the following moduli are also zero:

$$C'^{1112} = C'^{1222} = C'^{1233} = C'^{1323} = 0 \tag{9}$$

Primed stiffnesses relate to the rotated coordinate system. Equations (7–9) define the constitutive behavior of the composite material plies currently considered. The plate consists of a number of plies oriented at different angles and stacked together.

Assuming that the stress perpendicular to the plate surface in each ply is negligible,

$$T^{33} = 0 \tag{10}$$

the through-the-thickness strain is determined by Eqs. (7) and (8),

$$e_{33} = -\frac{C^{33\alpha\beta} e_{\alpha\beta}}{C^{3333}} \tag{11}$$

By putting this into Eq. (7), a constitutive relation of the following form is obtained:

$$T^{\alpha\beta} = \left(C^{\alpha\beta\gamma\delta} - \frac{C^{\alpha\beta 33} C^{\gamma\delta 33}}{C^{3333}} \right) e_{\gamma\delta} = Q^{\alpha\beta\gamma\delta} e_{\gamma\delta} \tag{12}$$

$$T^{\alpha 3} = C^{\alpha 3 kl} e_{kl} = 2C^{\alpha 3\beta 3} e_{\beta 3} \tag{13}$$

where Eq. (8) has been used. Equation (12), which defines the stiffness tensor $Q^{\alpha\beta\gamma\delta}$, is the same constitutive relation as for a material under plane stress conditions, i.e., when $T^{k3} = 0$.

An approximate strain energy of a laminated composite plate can now be obtained by integration through the plate thickness h, using the kinematics according to Eqs. (2) and (3),

$$\begin{aligned} W &= \frac{1}{2}\int_{-h/2}^{h/2} T^{ij} e_{ij}\, dx_3 \approx \frac{1}{2}\int_{-h/2}^{h/2} \left(T^{\alpha\beta} e_{\alpha\beta} + 2T^{\alpha 3} e_{\alpha 3} \right) dx_3 \\ &= \frac{1}{2}\int_{-h/2}^{h/2} \left(Q^{\alpha\beta\gamma\delta} e_{\alpha\beta} e_{\gamma\delta} + 4C^{\alpha 3\beta 3} e_{\alpha 3} e_{\beta 3} \right) dx_3 \\ &= \frac{1}{2} A^{\alpha\beta\gamma\delta} e^0_{\alpha\beta} e^0_{\gamma\delta} + B^{\alpha\beta\gamma\delta} e^0_{\alpha\beta} \kappa_{\gamma\delta} \\ &+ \frac{1}{2} D^{\alpha\beta\gamma\delta} \kappa_{\alpha\beta} \kappa_{\gamma\delta} + kA^{\alpha\beta} e_{\alpha 3} e_{\beta 3} \end{aligned} \tag{14}$$

The approximation in the beginning is due to assumed small (negligible) through-the-thickness stress T^{33}. The shear correction factor k, which is introduced to correct for nonconstant shear stress distribution through the plate thickness, is here assumed to take the same value as introduced by Reissner,

$$k = 5/6 \tag{15}$$

Formally, the last equality in Eq. (14) is true only for $k = 1$. Whitney[9,10] has elaborated on other expressions for shear correction factors and used two k factors. Currently, however, Eq. (15) is adopted.

The plate stiffnesses in Eq. (14) are

$$(A^{\alpha\beta\gamma\delta}, B^{\alpha\beta\gamma\delta}, D^{\alpha\beta\gamma\delta}) = \int_{-h/2}^{h/2} \left(1, x_3, x_3^2 \right) Q^{\alpha\beta\gamma\delta}\, dx_3 \tag{16}$$

$$A^{\alpha\beta} = 2\int_{-h/2}^{h/2} C^{\alpha 3\beta 3}\, dx_3 \tag{17}$$

With the preceding strain energy, the constitutive relation of the plate can be written

$$\begin{aligned} N^{\alpha\beta} &= A^{\alpha\beta\gamma\delta} e^0_{\gamma\delta} + B^{\alpha\beta\gamma\delta} \kappa_{\gamma\delta} \\ M^{\alpha\beta} &= B^{\alpha\beta\gamma\delta} e^0_{\gamma\delta} + D^{\alpha\beta\gamma\delta} \kappa_{\gamma\delta} \\ Q^{\alpha} &= kA^{\alpha\beta} e_{\beta 3} \end{aligned} \tag{18}$$

Now consider a plate with either kinematic $\bar{u}_i^*$ and θ_α^*, dynamic $T^{*\alpha}$, Q_n^*, and $M^{*\beta}$, or mixed boundary conditions on the boundary Γ. Then by the principle of virtual work,

$$\begin{aligned} &\int \left(N^{\alpha\beta} \delta e^0_{\alpha\beta} + M^{\alpha\beta} \delta\kappa_{\alpha\beta} + 2Q^{\alpha} \delta e_{3\alpha} \right) dS \\ &= \int p^i \delta\bar{u}_i\, dS + \int \left(T^{*\alpha} \delta\bar{u}_\alpha + Q_n^* \delta\bar{u}_3 + \varepsilon^\alpha_\beta M^{*\beta} \delta\theta_\alpha \right) d\Gamma \end{aligned} \tag{19}$$

where ε^α_β is the two-dimensional permutation tensor. The integration in the right side is performed in the undeformed configuration since infinitesimal surfaces in the deformed and the undeformed configurations have the same area within the current approximation, independent of whether the surface is parallel to the plate middle surface or if it is on the plate edge.

The following equilibrium equations and dynamic boundary conditions are generated by Eqs. (3), (4), and (19),

$$
\begin{aligned}
N^{\alpha\beta}{}_{,\beta} + p^{\alpha} &= 0 \\
M^{\alpha\beta}{}_{,\beta} - Q^{\beta} &= 0 \\
Q^{\alpha}{}_{,\alpha} + \left[\left(N^{\alpha\beta}\bar{u}_{3,\beta}\right)_{\alpha}\right] + p^{3} &= 0
\end{aligned}
\tag{20}
$$

and

$$
\begin{aligned}
N^{\alpha\beta} n_{\beta} &= T^{*\alpha} \\
M^{\alpha\beta} n_{\beta} &= M^{*\beta}\varepsilon^{\alpha}_{\beta} \\
Q^{\alpha} n_{\alpha} + \left[N^{\alpha\beta} \cdot n_{\alpha} \cdot \bar{u}_{3,\beta}\right] &= Q^{*n}
\end{aligned}
\tag{21}
$$

By neglecting quadratic terms in the strain, i.e., by letting

$$
e^{0}_{\alpha\beta} = \left(\bar{u}_{\alpha,\beta} + \bar{u}_{\beta,\alpha}\right)/2 \tag{22}
$$

in Eq. (3), the terms within square brackets in Eqs. (20) and (21) vanish, and a linear problem is obtained.

Expressing the Plate Stiffnesses Using Lamination Parameters—Old Results

This paragraph presents the results of Tsai and Pagano[1] concerning lamination parameters for laminated Kirchhoff plates. The different plies constituting the plate are generally rotated different angles about the x_3 axis. The $Q^{\alpha\beta\gamma\delta}$ stiffnesses are transformed as

$$
Q^{\alpha\beta\gamma\delta} = Q'^{\eta\mu\kappa\zeta} \frac{\partial x_{\alpha}}{\partial x'_{\eta}} \frac{\partial x_{\beta}}{\partial x'_{\mu}} \frac{\partial x_{\gamma}}{\partial x'_{\kappa}} \frac{\partial x_{\delta}}{\partial x'_{\zeta}} \tag{23}
$$

where, according to Fig. 2, for the rotation $\varphi = \varphi(x_3)$ of a ply about the x_3 axis,

$$
\begin{aligned}
x_1 &= x'_1 \cos\varphi - x'_2 \sin\varphi \\
x_2 &= x'_1 \sin\varphi + x'_2 \cos\varphi
\end{aligned}
\tag{24}
$$

Observe that φ generally varies with x_3. As an example of the stiffness transformation,

$$
\begin{aligned}
Q^{1111} &= Q'^{1111}c^4 + 2Q'^{1122}c^2s^2 + 4Q'^{1212}c^2s^2 + Q'^{2222}s^4 \\
&= U_1 + U_2\cos 2\theta + U_3\cos 4\theta
\end{aligned}
\tag{25}
$$

where $c = \cos\varphi$ and $s = \sin\varphi$. The material constants U_1–U_4 are

$$
\begin{aligned}
U_1 &= \left(3Q'^{1111} + 3Q'^{2222} + 2Q'^{1122} + 4Q'^{1212}\right)/8 \\
U_2 &= \left(Q'^{1111} - Q'^{2222}\right)/2 \\
U_3 &= \left(Q'^{1111} + Q'^{2222} - 2Q'^{1122} - 4Q'^{1212}\right)/8 \\
U_4 &= \left(Q'^{1111} + Q'^{2222} + 6Q'^{1122} - 4Q'^{1212}\right)/8
\end{aligned}
\tag{26}
$$

In Eq. (25) the facts that $Q'^{1112} = Q'^{1222} = 0$, derived from Eqs. (9) and (12), were used. All $Q^{\alpha\beta\gamma\delta}$ stiffnesses are determined similarly,

$$
\begin{aligned}
Q^{1111} &= U_1 + U_2\cos 2\varphi + U_3\cos 4\varphi, & Q^{1111} &= Q_{11}, \\
Q^{2222} &= U_1 - U_2\cos 2\varphi + U_3\cos 4\varphi, & Q^{1112} &= Q_{16}, \\
Q^{1122} &= U_4 - U_3\cos 4\varphi, & Q^{1122} &= Q_{12}, \\
Q^{1212} &= (1/2)(U_1 - U_4) - U_3\cos 4\varphi, & Q^{1212} &= Q_{66}, \\
Q^{1112} &= (1/2)U_2\sin 2\varphi + U_3\sin 4\varphi, & Q^{1222} &= Q_{26}, \\
Q^{1222} &= (1/2)U_2\sin 2\varphi - U_3\sin 4\varphi, & Q^{2222} &= Q_{22},
\end{aligned}
\tag{27}
$$

The tensor components are given to the left; the right-hand notation is common in the composite material society. Now, the plate stiffnesses can be written as follows:

$$
\begin{aligned}
A^{*1111} &= U_1 + U_2\xi^A_1 + U_3\xi^A_2, \\
A^{*2222} &= U_1 - U_2\xi^A_1 + U_3\xi^A_2, \\
A^{*1122} &= U_4 - U_3\xi^A_2, \\
A^{*1212} &= 1/2\,(U_1 - U_4) - U_3\xi^A_2, \\
A^{*1112} &= (1/2)\,U_2\xi^A_3 + U_3\xi^A_4, \\
A^{*1222} &= (1/2)\,U_2\xi^A_3 - U_3\xi^A_4,
\end{aligned}
\qquad
\begin{aligned}
A^{1111} &= A_{11}, & A^{1212} &= A_{66}, \\
A^{1112} &= A_{16}, & A^{1222} &= A_{26}, \\
A^{1122} &= A_{12}, & A^{2222} &= A_{22},
\end{aligned}
\tag{28}
$$

$$
\begin{aligned}
B^{*1111} &= U_2\xi^B_1 + U_3\xi^B_2, \\
B^{*2222} &= -U_2\xi^B_1 + U_3\xi^B_2, \\
B^{*1122} &= -U_3\xi^B_2, \\
B^{*1212} &= -U_3\xi^B_2, \\
B^{*1112} &= 1/2U_2\xi^B_3 + U_3\xi^B_4, \\
B^{*1222} &= 1/2U_2\xi^B_3 - U_3\xi^B_4,
\end{aligned}
\qquad
\begin{aligned}
B^{1111} &= B_{11}, & B^{1212} &= B_{66}, \\
B^{1112} &= B_{16}, & B^{1222} &= B_{26}, \\
B^{1122} &= B_{12}, & B^{2222} &= B_{22},
\end{aligned}
\tag{29}
$$

$$
\begin{aligned}
D^{*1111} &= U_1 + U_2\xi^D_1 + U_3\xi^D_2, \\
D^{*2222} &= U_1 - U_2\xi^D_1 + U_3\xi^D_2, \\
D^{*1122} &= U_4 - U_3\xi^D_2, \\
D^{*1212} &= 1/2(U_1 - U_4) - U_3\xi^D_2, \\
D^{*1112} &= (1/2)U_2\xi^D_3 + U_3\xi^D_4, \\
D^{*1222} &= (1/2)U_2\xi^D_3 - U_3\xi^D_4,
\end{aligned}
\qquad
\begin{aligned}
D^{1111} &= D_{11}, & D^{1212} &= D_{66}, \\
D^{1112} &= D_{16}, & D^{1222} &= D_{26}, \\
D^{1122} &= D_{12}, & D^{2222} &= D_{22},
\end{aligned}
\tag{30}
$$

The normalization is

$$
A^{*\alpha\beta\gamma\delta} = \frac{1}{h}A^{\alpha\beta\gamma\delta}, \quad B^{*\alpha\beta\gamma\delta} = \frac{4}{h^2}B^{\alpha\beta\gamma\delta}, \quad D^{*\alpha\beta\gamma\delta} = \frac{12}{h^3}D^{\alpha\beta\gamma\delta} \tag{31}
$$

These results were derived by Tsai and Pagano[1]; the various ξ are the 12 lamination parameters,

$$
\begin{aligned}
\xi^A_{(1,2,3,4)} &= \frac{1}{2}\int_{-1}^{1} (\cos 2\varphi, \cos 4\varphi, \sin 2\varphi, \sin 4\varphi)\, \mathrm{d}z^* \\
\xi^B_{(1,2,3,4)} &= \int_{-1}^{1} (\cos 2\varphi, \cos 4\varphi, \sin 2\varphi, \sin 4\varphi)\, z^*\, \mathrm{d}z^* \\
\xi^D_{(1,2,3,4)} &= \frac{3}{2}\int_{-1}^{1} (\cos 2\varphi, \cos 4\varphi, \sin 2\varphi, \sin 4\varphi)\, z^{*2}\, \mathrm{d}z^*
\end{aligned}
\tag{32}
$$

where $z^* = 2x_3/h$ is the normalized through-the-thickness coordinate. The material in all plies must be the same (intralaminar hybrids may be allowed, but interlaminar hybrids are generally not).

Expressing the Plate Stiffnesses Using Lamination Parameters—New Results

The plate shear stiffnesses are

$$
A^{\alpha\beta} = 2\int_{-h/2}^{h/2} C^{\alpha 3\beta 3}\, \mathrm{d}x_3 = 2\int_{-h/2}^{h/2} \frac{\partial x_{\alpha}}{\partial x'_{\gamma}} \frac{\partial x_{\beta}}{\partial x'_{\delta}} C'^{\gamma 3\delta 3}\, \mathrm{d}x_3 \tag{33}
$$

which by the rotation, Eq. (24), are transformed into

$$\begin{aligned}
A^{11} &= 2\int_{-h/2}^{h/2}(C'^{1313}c^2 + C'^{2323}s^2)\,\mathrm{d}x_3 \\
&= h\,(C'^{1313} + C'^{2323}) + (C'^{1313} - C'^{2323})\int_{-h/2}^{h/2}\cos 2\varphi\,\mathrm{d}x_3 \\
A^{22} &= 2\int_{-h/2}^{h/2}(C'^{1313}s^2 + C'^{2323}c^2)\,\mathrm{d}x_3 \\
&= h\,(C'^{1313} + C'^{2323}) - (C'^{1313} - C'^{2323})\int_{-h/2}^{h/2}\cos 2\varphi\,\mathrm{d}x_3 \\
A^{12} &= 2\int_{-h/2}^{h/2}(C'^{1313} - C'^{2323})\,sc\,\mathrm{d}x_3 \\
&= (C'^{1313} - C'^{2323})\int_{-h/2}^{h/2}\sin 2\varphi\,\mathrm{d}x_3
\end{aligned} \tag{34}$$

where $c = \cos\varphi$ and $s = \sin\varphi$ and the fact that $C'^{1323} = 0$, according to Eq. (9), was used. The integrals are recognized as two of the in-plane lamination parameters, and the relation between the plate shear stiffnesses and these parameters is

$$\begin{aligned}
A^{*11} &= (C'^{1313} + C'^{2323}) + (C'^{1313} - C'^{2323})\,\xi_1^A \\
A^{*22} &= (C'^{1313} + C'^{2323}) - (C'^{1313} - C'^{2323})\,\xi_1^A \\
A^{*12} &= (C'^{1313} - C'^{2323})\,\xi_3^A
\end{aligned} \tag{35}$$

where the normalization is

$$A^{*\alpha\beta} = \frac{1}{h}\,A^{\alpha\beta} \tag{36}$$

Thus, according to Eqs. (28–30) and (35), all plate stiffnesses needed for the Reissner-Mindlin type plate theory are expressed by the same lamination parameters as those used in classical lamination theory with Kirchhoff kinematics.

Apart from the four material parameters U_1–U_4, two new material parameters must be introduced, the shear stiffnesses C'^{1313} and C'^{2323}. For certain reinforcements, such as unidirectional, only one new material parameter is needed. Unidirectional plies are usually assumed to be transversely isotropic in the plane perpendicular to the x'_1 axis, (see Fig. 2); the following moduli are then dependent:

$$C'^{3333} = C'^{2222}, \qquad C'^{1133} = C'^{1122}, \qquad C'^{1313} = C'^{1212}$$

$$C'^{2323} = (C'^{2222} - C'^{2223})/2 \tag{37}$$

Then according to Eqs. (9), (12), (26), and (37),

$$C'^{1313} = Q'^{1212} = \frac{U_1 - U_4}{2} - U_3 \tag{38}$$

Summary and Conclusion

The usage of lamination parameters for layered composite material plates has been extended from Kirchhoff kinematics to shear deformable Reissner-Mindlin kinematics.

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